Bachelor of Commerce

Semester – II

Paper Code –

BUSINESS MATHEMATICS - II

B. COM (Semester-II)

BUSINESS MATHEMATICS-II

PAPER CODE:

Theory Paper Max Marks: 80

Time: 3Hrs **Internal marks: 20**

Unit I

Linear Programming-Formulation of LPP: Graphical method of solution; Problems relating to two variables including the case of mixed constraints; Cases having no solution, multiple solutions, unbounded solution and redundant constraints.

Unit-II

Simplex Method—Solution of problems up to three variables, including cases of mixed constraints; Duality; Transportation Problem.

Unit-III

Compound Interest: Certain different types of interest rates; Concept of present value and amount of a sum

Unit-IV

Annuities: Types of annuities; Present value and amount of an annuity, including the case of continuous compounding; Valuation of simple loans and debentures; Problems relation to sinking funds.

Suggested Readings:

- Allen B.G.D: Basic Mathematics; Mcmillan, New Delhi.
- Volra. N. D. Quantitative Techniques in Management, Tata McGraw Hill, New Delhi. Kapoor V.K. Business
- Mathematics: Sultan chand and sons, Delhi.

Contents

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Linear Programming

Structure

1.1. Introduction. For manufacturing a product, a number of resources like raw material, machines, manpower and other types of materials. These resources are available in limited quantity but their demand is more. So they are used be optimally utilized so that cost could be minimized and profits could be maximized. Linear programming is a technique which helps us in the optimal utilization of these resources.

1.1.1. Objective. The objective of these contents is to provide some important results to the reader like:

(i) Formulation of LPP.

(ii) Graphical method of solution.

(iii) Cases having no solution, multiple solutions, unbounded solution and redundant constraints.

1.1.2. Keywords. Linear Programming, Unique Solution, Unbounded Solution.

1.2. Linear Programming. A linear programming problem consists of a linear function to be maximized or minimized subject to certain constraints in the form of linear equations or inequalities.

1.2.1. Characteristics of Linear Programming

1. Every linear programming problem has an objective which should be clearly identifiable and measurable. For example objective can be maximization of sales, profits and minimization of costs and so on.
2. All the products and resources should also be clearly identifiable and measurable.
3. Resources are available in limited quantity.
4. The relationship representing objectives and resources limitation are represented by constraint in equalities or equations. These relationships are linear in nature.

The above characteristics will be clear from the following example:

1.2.2. Example. A firm is engaged in manufacturing two products A and B. Each unit of A requires 2 kg. of raw material and 4 labour hours for processing while each unit of B requires 3 kg. of raw material and 3 labour hours. The weekly availability of raw material and labour hours is limited to 60 kg. and 96 hours respectively. One unit of product A sold for Rs. 40 while are unit of B is sold for Rs. 35.

In the above problem, first we define the objective. Since we are given data on sales prince per unit of two products, so our objective is to maximize the soles.

Let x_1 be the number of units of A and x_2 be the number of units of B to be produced. So Sales revenue from sale of x_1 unit of A = $40x_1$ and sales revenue from sale of x_2 units of B = $35x_2$. Thus

$$\text{Total sales} = 40x_1 + 35x_2 \text{ and}$$

Our objective becomes

$$\text{Maximise } Z = 40x_1 + 35x_2$$

After this we come to the constraint in equalities, from the given date, we find that quantity of raw material required to produce x_1 kg of A = $2x_1$ [each unit of A requires 2 kg of raw material so x_1 units of A require $2x_1$ kg of raw material]

In the same way, quantity of raw material required to produce x_2 units of B = $3x_2$.

So new total raw material requirement = $2x_1 + 3x_2$

But here we have a constraint in the form of quantity of raw material available. Since the maximum quantity of raw material available is limited to 60 kg, so we cannot use more than 60 kg of raw material in any case. Mathematically we can write it as

$$2x_1 + 3x_2 \leq 60$$

In other words we can say that total quantity of raw material consumed is less than and equal to (shown by the symbol $\leq$) 60

Similarly we can produced to expressed the labour constraints in the following way:

Labour hours required to produce to produce x_1 units of A = $3x_1$ and

Labour hours required x_2 units of B = $3x_2$.

Since the total number of labour hours available per week is limited to 96, we can express this constraint as

$$3x_1 + 3x_2 \leq 96$$

In the last we express the non-negativity condition, i.e. $x_1, x_2 \geq 0$

Which is self-clear because number of units of product A or B can be zero or positive only.

Now the above problem can be summarised as

$$\text{Max. } Z = 40x_1 + 35x_2$$

Subject to

$$2x_1 + 3x_2 \leq 60$$

$$3x_1 + 3x_2 \leq 96$$

$$x_1, x_2 \geq 0.$$

Generally we can express a linear programming in the following way:

$$\text{Maximise or Minimise } Z = C_1x_1 + C_2x_2 + \ldots + C_nx_n \quad \text{objective function}$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \text{ or } \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \text{ or } \geq b_2$$

$$\ldots \ldots \ldots \ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_n \text{ or } \geq b_n$$

$$x_1, x_2, x_3, \ldots, x_n \geq 0$$

where C's are the profit or cost coefficients of decisions variables x's, a's are the resource coefficients and b's are the resource values.

1.3. Solutions of Linear Programming Problems.

There are two methods of solving the linear programming problems - Graphical method and Simplex method.

1.3.1. Graphical Method.

By this method, we can solve problems involving two variables only, one variable is taken as x and the second as y. On any graph we can show only two variables. The steps involved in this method are following:

1. We draw a horizontal straight line on a graph paper which is called x-axis.
2. We draw a vertical straight line which is perpendicular to this horizontal line. This perpendicular line is called Y-axis.
3. Constraints inequalities are converted into equations and plot their points on the graph.
4. Common feasible area of all the inequalities is found out. From this area, we get the maximum or minimum values of Z.

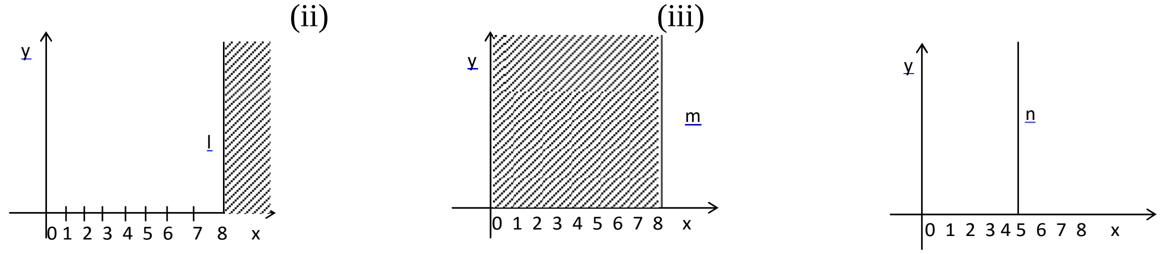
The following illustrations will make it more clear:

1.3.2. Example. Find the feasible area of the following :

(i) $x \leq 8$ (ii) $x \geq 6$ (iii) $x = 5$

Solution.

(i) (ii) (iii)



(Shaded portion shows the feasible area)

In the first case, feasible area is to the right of the vertical line *l*. There is no limit to its maximum value. In the second case, the feasible area is to the left of the line m forwards y-axis and in third case, feasible area is along the line n.

To find feasible area of an equation having two variables only.

1.3.3. Example. Find the feasible area of the following :

(i) $2x + y \leq 6$ (ii) $2x + y \leq 6$, $x, y \geq 0$.

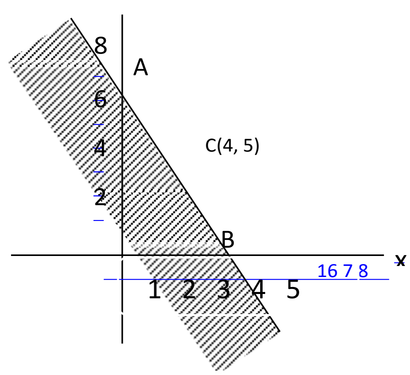
Solution (i) For plotting the lines, change the inequalities into equations so $2x+y=6$

Find two points to be plotted on the graph

When $x = 0$ $y = 6$

When $y = 0$ $x = 3$

So we get two points (0, 6) and (3, 0) which when plotted on the graph and joined by a line gives us the line of the above equation.

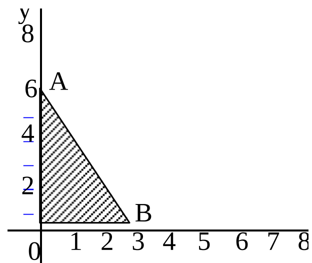


Now since there is no mention of the signs of x and y. So feasible area extends infinitely to the left of line joining points. A(0,6) and B(3, 0). Any point taken to the right of line joining A and B does not satisfy this condition of inequality. For example, let us consider point C(4, 5) Substituting the values of x = 4 and y = 5 in the L.H.S. of the inequality we get

$$2x + y = 2 * 4 + 5 = 13$$

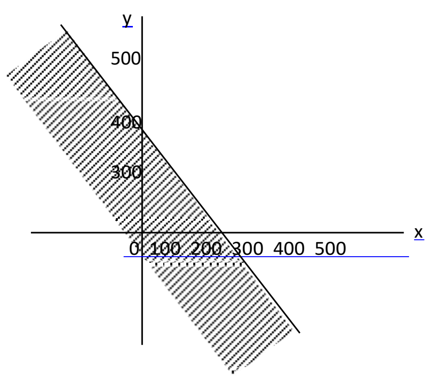
which is not less than 6. Hence this point does not lies in the feasible area of the in equation.

(ii) In this case, since we are given x, y ≥ 0, it means that neither x nor y can have negative sign. So the feasible area of the inequality 2x + y ≤ 6 lies in the region OAB.



1.3.4. Example. Draw the graph of the in equation $\frac{x}{200} + \frac{y}{300} \leq 1$. Which of the following points lie in the graph (i) 300, 0 (ii) 200, 400 (iii) 150, 250

Solution. Given $\frac{x}{200} + \frac{y}{300} \leq 1$



or $3x + 2y \leq 600$

Changing into equation

$$3x + 2y = 600$$

When x = 0, y = 300

y = 0, x = 200

From the graph, it is clear that

(i) Point (300, 0) does not lie in the graph.

(ii) Point (200, 400) also does not lie in graph but

(iii) Point (150, 250) lies in the graph.

13.5. Example. Draw the diagram of solution set of the linear constraints

$$2x + 3y \leq 6$$

$$x + 4y \leq 4$$

$$x \geq 0, y \geq 0.$$

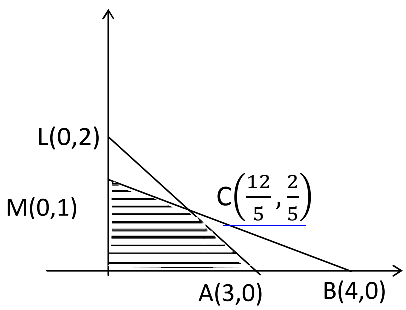
Solution. The given constraints are

$$2x + 3y \leq 6$$

$$x + 4y \leq 4$$

$$x \geq 0, y \geq 0.$$

Consider a set of rectangular cartesian axes OXY in the plane. Each point has co-ordinates of the type (x, y) and conversely. It is clear that any point which satisfies $x \geq 0, y \geq 0$ lies in the first quadrant.



Let us draw the graph of $2x + 3y = 6$

For $x = 0, 3y = 6,$ i.e., $y = 2$

For $y = 0, 2x = 6$ i.e., $x = 3$

Therefore, line $2x + 3y = 6$ meets OX in A(3, 0) and OY in L(0, 2)

Again let us draw the graph of

$$x + 4y = 4$$

For $x = 0, 4y = 4$ i.e., $y = 1$

For $y = 0, x = 4$

Therefore, line $x + 4y = 4$ meets OX in B(4, 0) and OY in M(0, 1).

Since feasible region is the region which satisfies all the constraints, feasible region is the quadrilateral OACM.

The corner points are O(0, 0), A(3, 0), C(12/5, 2/5), M(0, 1).

Note

1. Students must note that graphs are to be drawn on the graph paper.

2. Point C can also be calculated by solving the equation $2x + 3y = 6$, $x + 4y = 4$. It helps them in verifying the result obtained from the graph.

3. $2x + 3y \leq 6$ represents the region on and below the line AL. Similarly $2x + 3y \geq 6$ will represent the region on and above the line AL.

4. In order to have a clear picture of the region below or above the line, it is better to note that if the point (x_1, y_1) satisfies the in equation then the region containing this point is the required region.

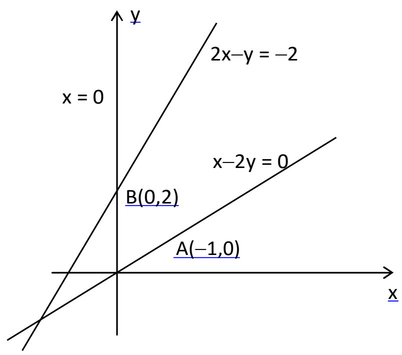
1.3.6. Example. Verify that the solution set of the following linear constraints is empty:

$$x - 2y \geq 0,\ 2x - y \leq -2,\ x \geq 0,\ y \geq 0.$$

Solution. The straight line $x - 2y = 0$ passes through O.

The straight line $2x - y = -2$ meets x-axis at A(-1, 0) and y-axis at B(0, 2).

We have the following figure.



Since no portion satisfies all the four constraints,

Therefore, the solution set is empty.

1.4. Solution of LPP by graphical method.

1.4.1. Maximisation case.

1.4.2. Example. A furniture dealer deals in only two items : tables and chairs. He has Rs. 5000.00 to invest and a space to store at most 60 pieces. A table costs him Rs. 250.00 and a chair Rs. 50.00. He can sell a table at a profit of Rs. 50.00 and a chair at a profit of Rs. 15.00. Assuming that he can sell all the items that he buys, how should he invest his money in order that he may maximize his profit ?

Solution. We formulate the problem mathematically.

Max. possible investment = Rs. 5000.00

Max. storage space = 60 pieces of furniture

	Cost	Profit
Table :	Rs. 250.00	Rs. 50.00
Cost Chair :	Rs. 50.00	Rs. 15.00

Let x and y be the number of tables and chairs respectively. Then we have the following constraints :

$x \geq 0$ ...(1) $y \geq 0$ ...(2)

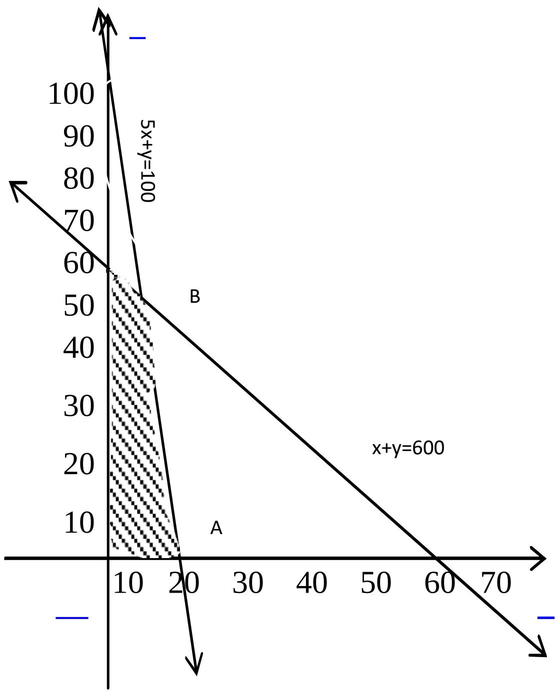
$250x + 50y \leq 5000$ i.e. $5x + y \leq 100$ ...(3)

and $x + y \leq 60$ ...(4)

Let Z be the profit, then $Z = 50x + 15y$ ...(5)

We are to maximize Z subject to constraint (1), (2) (3) and (4).

Let us graph the constraints given in (1), (2), (3) and (4).



Explanation. Draw the straight lines x = 0, (y-axis), y = 0 (x-axis).

Draw the straight line x + y = 60. This meets the x-axis at (60, 0) and y-axis at (0, 60).

Draw the straight line 5x + y = 100. This meets x-axis at (20, 0) and y-axis at (0, 100).

The shaded region consists of points, which are the intersections of four constraints. This region is called feasible solution of the linear programming problem.

The vertices of the figure OABC show the possible combinations of x and y one of which gives us the maximum value. Now we consider the points one by one.

1.4.3. Example. A company manufactures two types of telephone sets, one of which is cordless. The cord type telephone set requires 2 hours to make, and the cordless model requires 4 hours. The company has at most 800 working hours per day to manufacture these models and the packing department can pack at the most 300 telephone sets per day. If the company sells, the cord type model for Rs. 300 and the cordless model for Rs. 400, how many telephone sets of each type should it produce per day to maximize its sales ?

Solution. Let the number of cord type telephone sets be x and the number of cordless type telephone sets be y.

Clearly we have : $x \geq 0$; $y \geq 0$ …(i)

Also $2x + 4y \leq 800$ …(ii)

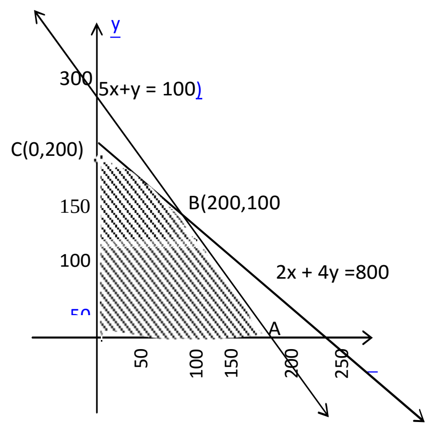
$x + y \leq 300$ …(iii)

Let Z be sale function, Z = 300 x + 400 y

The problem reduces to maximize sale function subject to the conditions : $x \geq 0$, $y \geq 0$

$2x + 4y \leq 800$, $x + y \leq 300$ …(A)

Let us draw the graph of system (A) and solution set of these inequations is the shaded region OABC and so the feasible region is the shaded region whose corner points are O(0, 0), A(300, 0), B(200, 100), C(0, 200). Since the maximum value of the sale function occurs only at the boundary point (s) and so we calculate the sale function at every point of the feasible region.



Boundary points of the feasible region	S = 300 x + 400 y
O(0, 0)	S = 300*0 + 400*0 = 0
A(300, 0)	S = 300*300 + 400*0 = Rs. 90, 000
B(200, 100)	S = 300*200 + 400*100 = Rs. 100000
C(0, 200)	S = 300*0 + 400*200 = Rs. 80, 000

Hence the maximum sale is Rs. 100000 at B(200, 100) and so company should produce 200 cord type and 100 cordless telephone sets.

1.4.4. Example. Solve the following LPP

Maximize Z = 10x + 12y

Subject to the constraints :

$x + y \leq 5$

$4x + y \geq 4$

$x + 5y \geq 5$

$x \leq 4$

$y \leq 3$.

Solution. It is a problem of mixed constraints. Constraints having greater than or equal to (≥) sign will have their feasible area to the right of their line while the constraints having less than a equal to (≤) sign will have their area to the left of their line.

The given linear constraints are :

$$x + y \leq 5 \qquad \ldots(1)$$

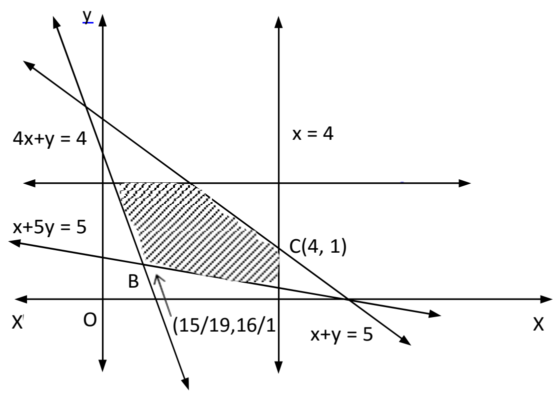
$$4x + y \geq 4 \qquad \ldots(2)$$

$$x + 5y \geq 5 \qquad \ldots(3)$$

$$x \leq 4 \qquad \ldots(4)$$

$$y \leq 3 \qquad \ldots(5)$$

Let us draw the graph of inequations (1), (2), (3), (4) and (5). The graph (or solution set) of these inequations is the shaded area (a polygen ABCDE) with vertices (1/4,3), (15/19, 16/19), (4,1/5), (4, 1), (2, 3) as shown in the figure below :



This shaded area is bounded by the five lines x + y = 5, 4x + y = 4, x + 5y = 5, x = 4 and y = 3.

Boundary points of the feasible region	Z = 10x + 12y
$A\left(\frac{1}{4}, 3\right)$	$Z = 10 \times \frac{1}{4} + 12 \times 3 = 38.5$
$B\left(\frac{15}{19}, \frac{16}{19}\right)$	$Z = 10 \times \frac{15}{19} + 12 \times \frac{16}{19} = 18$
$C\left(4, \frac{1}{5}\right)$	$Z = 10 \times 4 + 12 \times \frac{1}{5} = 42.4$
D(4,1)	$Z = 10 \times 4 + 12 \times 1 = 52$
E(2,3)	$Z = 10 \times 2 + 12 \times 3 = 56$

Since maximum value of Z is Rs. 56 at E, so optimum solution is x = 2, y = 3.

1.4.5. Example. If a young man rides his motor-cycle at 25 km per hour, he has to spend Rs. 2 per km on petrol, if he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what the maximum distance he can travel within one hour is? Express this as a linear programming problem and then solve it.

Solution. Let the young man ride x km at the speed of 25 km per hour and y km at the speed of 40 km per hour. Let f be the total distance covered, which is to be maximized.

Therefore, $f = x + y$ is the objective function.

Cost of travelling per km is Rs. 2 at the speed of 25 km per hour and cost of travelling per km is Rs. 5 at the speed of 40 km per hour.

Therefore, total cost of travelling = $2x + 5y$

Also Rs. 100 are available for petrol. Therefore,

$$2x + 5y \leq 100$$

Time taken to cover x km at the speed of 25 km per hour = $\frac{x}{25}$ hour

Time taken to cover y km at the speed of 40 km per hour = $\frac{y}{40}$ hour

Total time available = 1 hour

Therefore, we have

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

or $$8x + 5y \leq 200$$

Also $x \geq 0,\ y \geq 0$

Therefore, we are to maximize

$$f = x + y$$

subject to the constraints

$$2x + 5y \leq 100$$

$$8x + 5y \leq 200$$

$$x \geq 0\ ,\ y \geq 0.$$

Consider a set of rectangular cartesian axes OXY in the plane.

It is clear that any point which satisfies $x \geq 0,\ y \geq 0$ lies in the first quadrant.

Let us draw the graph of the line $2x + 5y = 100$

For $x = 0$, $5y = 100$ or $y = 20$

For $y = 0$, $2x = 100$ or $x = 50$

Therefore, line meets OX in A(50, 0) and OY in L(0, 20)

Again we draw the graph of the line

$$8x + 5y = 200.$$

For x = 0, 5y = 200 or y = 40

For y = 0, 8x = 200 or x = 25

Therefore, line meets OX in B(25, 0) and OY in M(0, 40).

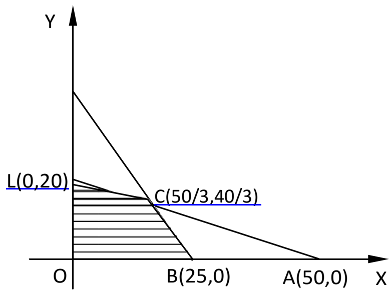
Since feasible region is the region which satisfies all the constraints,

Therefore, feasible region is the quadrilateral OBCL. The corner points are O(0, 0), B(25, 0), C(50/3,40,3), L(0, 20)

At O(0, 0) f = 0 + 0 = 0

At B(25, 0) f = 25 + 0 = 25

At C(50/3,40/3) f = 50/3 + 40/3 = 30



At L(0, 20) f = 0 + 20 = 20

Therefore, maximum value of f = 30 at (50/3,40/3).

Thus,the young man covers the maximum distance of 30 km when he rides 50/3 km at the speed of 25 km per hour and 40/3 km at the speed of 40 km per hour.

1.4.6. Example. A farmer decides to plant up to 10 hectares with cabbages and potatoes. He decided to grow at least 2, but not more than 8 hectares of cabbage and at least 1, but not more than 6 hectares of potatoes. If he can make a profit of Rs. 1500 per hectare on cabbages and Rs. 2000 per hectare on potatoes how should he plan his farming so as to get the maximum profit ? (Assuming that all the yield that he gets is sold.)

Solution. Suppose the farmer plants x hectares with cabbages and y hectares with potatoes.

Then the constraints are

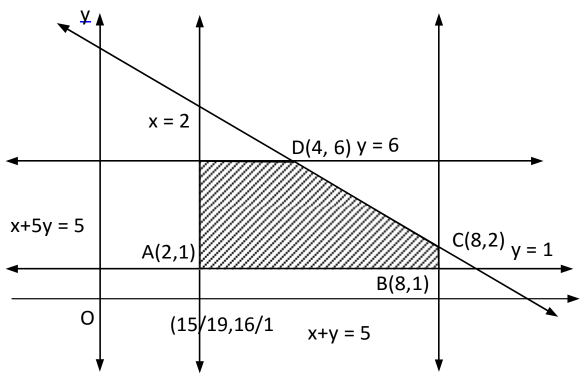
$2 \leq x \leq 8$ …(1)

$1 \leq y \leq 6$ …(2)

$x + y \leq 10$ …(3)

and $P = 1500x + 2000y$ …(4)

We draw the lines x = 2, x = 8, y = 1, y = 6 and x + y = 10.



The vertices of the solution set ABCDE are

A(2, 1), B(8, 1), C(8, 2), D(6, 4) and E(2, 6)

Now

at A(2, 1), P = 1500 (2) + 2000(1) = 3000 + 2000 = 5000

at B(8, 1), P = 1500 (8) + 2000(1) = 12000 + 2000 = 14000

at C(8, 2), P = 1500 (8) + 2000(2) = 12000 + 4000 = 16000

at D(4, 6), P = 1500 (4) + 2000(6) = 6000 + 12000 = 18000

at E(2, 6), P = 1500 (2) + 2000(6) = 3000 + 12000 = 15000.

Hence in order to maximise profit the farmer plants 4 hectares with cabbages and 6 hectares with potatoes.

1.4.7. Example. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by economy class than by the first class. Determine how many each type of tickets must be sold in order to maximise the profit for the airline. What is the maximum profit ?

Solution. Let the number of first class tickets and Economy class tickets sold by the Airline be x and y respectively.

Maximum capacity of passengers is 200 i.e. $x + y \leq 200$ …(i)

At least 20 seats of first class are reserved $x \geq 20$ …(ii)

At least 4 x seats of Economy class are reserved $y \geq 4x$ …(iii)

Let P the profit function, $P = 400x + 300y$

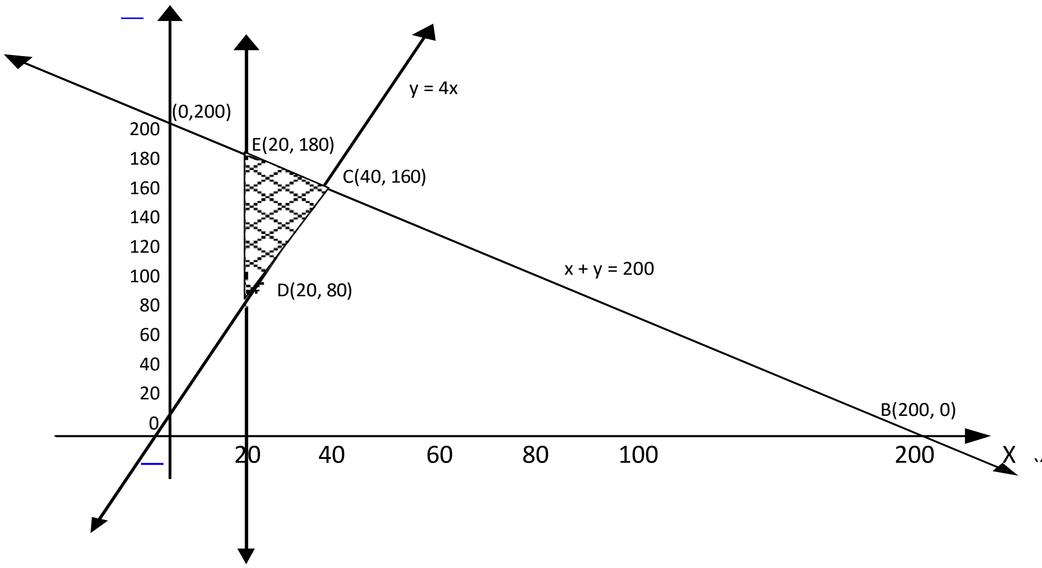
Therefore, the problem reduces to maximize P subject to the constraints $x \geq 20$; $y \geq 4x$ and $x + y \leq 200$.

Let us find out the solution set of the inequations $x \geq 20$; $y \geq 4x$ and $x + y \leq 200$.

The triangular shaded region CDE is the feasible region and its vertices are :

C(40, 160), D(20, 80), E(20, 180)

Since the maximum or minimum value occurs at the boundary point (s) and we calculate the profit function P at every vertex of the feasible region.



Boundary points of the feasible region	P = 400x + 300y
C(40, 160)	P = 400*40 + 300*160 = Rs. 64,000
D(20, 80)	P = 400*20 + 300*80 = Rs. 32,000
E(20, 180)	P = 400*20 + 300*180 = Rs. 62,000

Therefore, the maximum profit Rs. 64000 is obtained at C(40, 160) and so Airline should sell 40 tickets of first and 160 tickets of economy class.

1.4.8. Minimisation Case.

In such questions, value of the objective function is to be minimised. Generally, in the questions involving cost, distance, expenses risk etc. our objective to keep their value least.

Generally in case of maximisation, we use the constraints of less than or equal to (≤) type and in case of minimisation, we use constraints of greater than a equal to (≥) type. But same some times, we also use mixed constraints (both ≥ and ≤).

1.4.9. Example. Minimize

$$C = x + y$$

subject to

$$3x + 2y \geq 12$$

$$x + 3y \geq 11$$

$$x \geq 0$$

$$y \geq 0$$

Solution. Let us solve graphically the following inequations

$3x + 2y \geq 12$

$x + 3y \geq 11$

$x \geq 0$

$y \geq 0$

Changing the inequalities into equations, we get

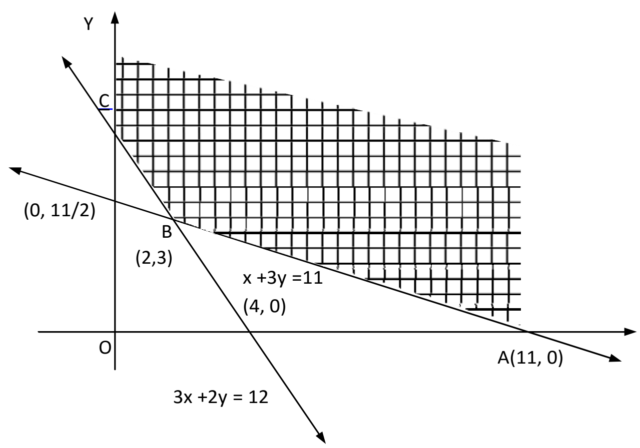
$3x + 2y = 12$	$x + 3y = 11$
For x=0, y=6	For x=11, y=0
For x=4, y=0	For x=0, y=11/3

Shaded region ABC is the required feasible region of the above stated inequations.

Boundary points of the feasible region are :

A(11, 0), B(2, 3), C(0, 6).

Since minimum or maximum always occurs only at boundary point (s) and so will calculate the cost (C) at every boundary point of the feasible region.



Boundary point of the feasible region	C = x + y
A(11, 0)	11 +0 = Rs. 11
B(2, 3)	2 + 3 = Rs. 5 (Mini. Cost)
C(0, 6)	0 + 6 = Rs. 6

Hence minimum cost is at the point B(2, 3) and minimum cost is Rs. 5.

1.4.10. Example. Find the maximum and minimum values of the function 2 = 3x + y

Subject to the constraint

$x + y \leq 2$

$4x + y \leq 5$

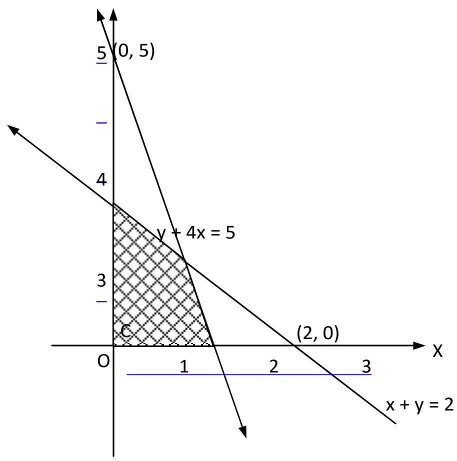
$x, y \geq 0$

Solution. Let us first change the inequalities into equation

x + y = 2	4x + y = 5
For x=0, y=2	For x=0, y=5
For x=2, y=0	For x=5/4, y=0

Solution set of the above inequations is the required shaded feasible region OABC whose boundary points are O(0, 0), A(5/4,0), B(1, 1), C(0, 2).

Since the minimum or maximum value of C occurs only at the boundary point (s) and so let us calculate the value of Z at every vertex of the feasible region OABC.



Boundary points of the feasible region	Z = 3x + y
O(0, 0)	Z = 3* 0 + 0 = Re. 0 (Minimum cost)
A(5/4 , 0)	Z = 3*(5/4) + 0 = Rs. 3.75
B(1, 1)	Z = 3*1 + 1 = Rs. 4
C(0, 2)	Z = 3*0 + 2 = Rs. 2

So the maximum value is Rs. 4 and the minimum value Rs. 0 for maximisation x = 1, y = 1 and for minimization x = 0, y = 0.

1.4.11. Example. Minimise P = 2x + 3y, subject to the conditions

$$x \geq 0, y \geq 0, 1 \leq x + 2y \leq 10.$$

Solution. We have

$x \geq 0$	…(1)	$y \geq 0$	…(2)
$x + 2y \geq 1$	…(3)	$x + 2y \leq 10$	…(4)

and P = x + 3y

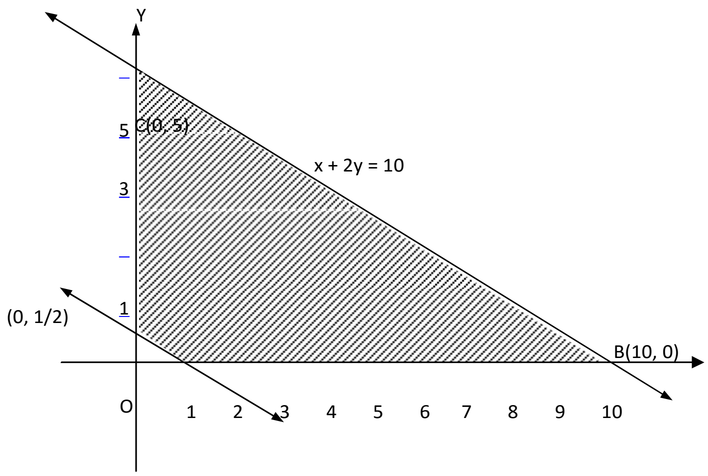
We find out the solution set (convex polygon), where (1) - (4) are true.

For this, we draw the graph of the lines x = 0, y = 0, x + 2y = 1, x + 2y = 10.

The shaded portion is the feasible region of the constraints.

Now

at A(1, 0),	P = 2(1) + 3(0) = 2
at B(10, 0),	P = 2(10) + 3(0) = 20
at C(0, 5),	P = 2(0) + 3(5) = 15
at D(0,1/2),	P = 2(0) + 3(1/2) = 3/2



Since the minimum value is at D, so the optimal solution is x = 0, y =1/2.

1.4.12. Example. Find the maximum and minimum value of

$$Z = x + 2y$$

subject to

$$2x + 3y \leq 6$$

$$x + 4y \leq 4$$

$$x, y \geq 0$$

Solution. We are maximize and minimize

$$Z = x + 2y$$

Subject of the constraints $2x + 3y \leq 6$

$$x + 4y \leq 4$$

$$x, y \geq 0$$

First, we draw the graph of the line 2x + 3y = 6.

For x = 0, 3y = 6, or y = 2

For y = 0, 2x = 6, or x = 3

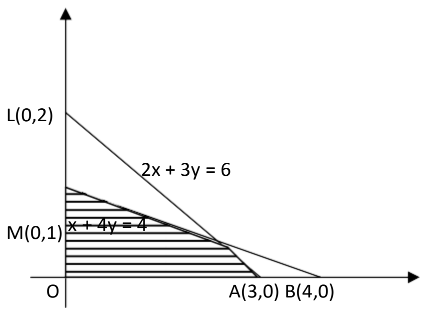
Therefore, line meets OX in A(3, 0) and OY in L(0,2).

Draw the graph of line x + 4y = 4

For x = 0, 4y = 4 or y = 1

For y = 0, x = 4

Therefore, line meets OX in B(4, 0) and OY in M(0,1).



Since feasible region is the region which satisfies all the constraints. OACM is the feasible region. The corner points are O(0, 0), A(3, 0), C(12/5, 2/5), M(0, 1).

At O(0, 0),	f = 0 + 0 = 0
At A(3, 0),	f = 3 + 0 = 3
At C(12/5, 2/5),	f = 12/5 + 4/5 = 16/5 = 3.2
At D(0,1),	f = 0+2 =2

Therefore, minimum value = 0 at (0,0) and maximum value = 3.2 at (12/5, 2/5).

1.5. Check Your Progress.

Draw the diagrams of the solution sets of the following (1 - 3) linear constraints :

1. $3x + 4y \geq 12$, $4x + 7y \leq 28$, $x \geq 0$, $y \geq 1$.
2. $x + y \leq 5$, $4x + y \geq 4$, $x + 5y \geq 5$, $x \leq 4$, $y \leq 3$.
3. $x + y \geq 1$, $y \leq 5$, $x \leq 6$, $7x + 9y \leq 63$, $x, y \geq 0$.
4. Draw the graph of the equation $2x + 3y \leq 35$.

5. Verify that the solution set of the following constraints is empty : $3x + 4y \geq 12$, $x + 2y \leq 3$, $x \geq 0$, $y \geq 1$.

6. Verify that the solution set of the following constraints : $x - 2y \geq 0$, $2x - y \leq -2$ is not empty and is unbounded.

7. Draw the diagram of the solution set of the linear constraints

(i)	(ii)
$3x + 2y \leq 18$	$2x + y \geq 4$
$x + 2y \leq 10$	$3x + 5y \geq 15$
$x \geq 3$, $y \geq 0$	$x \geq 0$, $y \geq 0$

8. Exhibit graphically the solution set of the linear constraints

$x + y \geq 1$

$y \leq 5$

$x \leq 6$

$7x + 9y \leq 63$

$x, y \geq 0.$

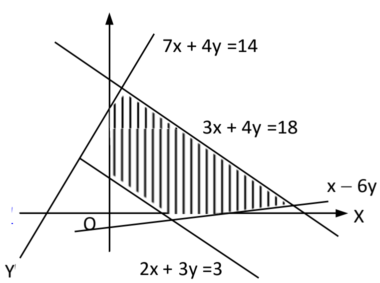
9. Verify that the solution set of the following linear constraints is empty :

(i) $x - 2y \geq 0$, $2x - y \leq -2$, $x \geq 0, y \geq 0$

(ii) $3x + 4y \geq 12$, $x + 2y \leq 3$, $y \geq 1, x \geq 0$

10. Verify that the solution set of the following linear constraints is unbounded :

$3x + 4y \geq 12,\ y \geq 1,\ x \geq 0$



11. Find the linear constraints for which the shaded area in the figures below is the solution set :

12. Find the maximum and minimum value of $2x + y$ subject to the constraints

$2x + 3y \leq 30,\ x - 2y \leq 8,\ x \geq 0,\ y \geq 0.$

13. Solve by graphical method :

(i) Minimize $Z = 3x_1 + 2x_2$ subject to the constrains

$-2x_1 + x_2 \leq 1,\ x_1 \leq 2,\ x_1 + x_2 \leq 3,\ x_1, x_2 \geq 0.$

(ii) Find the maximum value of $z = 5x_1 + 3x_2$ subject to the constraints

$3x_1 + 5x_2 \leq 15,\ 5x_1 + 2x_2 \leq 10,\ x_1, x_2 \geq 0.$

(iii) Find the maximum value of $z = 2x_1 + 3x_2$ subject to the constraints

$x_1 + x_2 \leq 1,\ 3x_1 + x_2 \leq 4,\ x_1, x_2 \geq 0.$

(iv) Find the minimum value of $3x + 5y$ subject to the constraints

$-2x + y \leq 4,\ x + y \geq 3,\ x - 2y \leq 2,\ x, y \geq 0.$

14. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space of at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming he can sell all the items that he can buy, how should he invest his money in order to maximise his profit ? Also find the maximum profit.

15. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg. of vitamin A and I unit per kg. of vitamin C, while the food II contains 1 unit per kg. of Vitamin A and 2 units per kg. of vitamin C. It costs Rs. 5 per kg. to purchase food I and Rs. 7 per kg. to purchase food II Find the minimum cost of such mixture and the quantity of the each of the foods.

15. A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine A and 3 on machine B to produce a package of nuts while it takes 32 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 2.50 per package on nuts and Rs. 1.00 per package on bolts. How many packages of each should he produce each day so as maximise his profit, if he operates his machines for at the most 12 hours a day ?

16. A sports factory prepares cricket bats and hockey sticks. A cricket nat bat takes 2 hours of machine time and 3 hours of craftsman’s time. A hockey stick take 3 hours of machine time and 2 hours of craftsman’s time. The factory has 90 hours of machine time and 85 hours of craftsman’s time. What number of bats and sticks must be made if the factory is to work at full capacity ? If the profit on a bat is Rs. 3 and on a stick it is Rs. 4, find the maximum profit.

17. A trader deals in sewing machines and transistors. It has capacity to store at the most 30 pieces and he can invest Rs. 4500. A machine costs him Rs. 250 each and transistor costs him Rs. 100 each. The profit on machine Rs. 40 and on a transistor it is Rs. 25. Find number of sewing machines and transistors to take max. profit.

18. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 2 per kg and rice Rs. 8. The minimum daily requirements of protein and carbohydrates for an average child are 50 g and 200 g respectively. In what quantities should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost.

19. A toy company manufactures two types of dolls, a basic version-doll A and a deluxe version-doll B. Each doll of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2,000 per day if it produced only the basic version. The supply of plastic is sufficient t o produce 1,500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs. 3.00 and Rs. 5.00 per doll respectively on doll A and B ; how many of each should be produced pe r day in order to maximize profit ?

20. Smita goes to the market to purchase battons. She needs at least 20 large battons and at least 30 small battons. tHe The shopkeeper sells battons in two forms (i) boxes and (ii) cards. A box contains then large and five small battons and a card contains two large and five small battons. Find the most economical way in which she should purchase the battons, if a box costs 25 paise and a card 10 paise only.

21. Vikram has two machines with which he can manufacture either bottles or tumblers. The first of the two machines has to be used for one minute and the second for two minutes in order to manufacture a bottle and the two machines have to be used for one minute each to manufacture a tumbler. During an hour the two machines can be operated for at the most 50 and 54 minutes respectively. Assuming that he can sell as many bottles and tumblers as the can produce, find how many of bottles and tumblers he should manufacture so that his profit per hour is maximum being given that the gets a profit of ten paise per bottle and six paise per tumbler.

22. A company produces two types of presentation goods A and B that require gold and silver. Each unit of type A requires 3 gms. of silver and 1 gm of gold while that of B requires 1 gm. of silver and 2 gm. of gold. The company can produce 9 gms. of silver and 8 gms. of gold. If each unit of type A brings a profit of Rs. 40 and that of type B Rs. 50 determine the number of units of each type that the company should produce to maximize the profit. What is the maximum profit ?

23. A gardener uses two types of fertilizers I and II. Type I consists of 10% nitrogen and 6% phosphoric acid while type II consists of 5% nitrogen and 10% phosphoric acid. He requires at least 14 kg. of both nitrogen and phosphoric acid for his crop. If the type I fertilizer costs Rs. 0.60 per kg and type II costs Rs. 0.40 per kg., how many kilograms of each fertilizer he should use so as to minimise the total cost. Also find the minimum cost.

1.6. Summary. In this chapter, we discussed about solving the given maximization and minimization problems using graphical solutions and observed that there are variety of regions in which solution exists and sometimes does not exists.

Books Suggested.

1. Allen, B.G.D, Basic Mathematics, Mcmillan, New Delhi.
2. Volra, N. D., Quantitative Techniques in Management, Tata McGraw Hill, New Delhi.
3. Kapoor, V.K., Business Mathematics, Sultan chand and sons, Delhi.